Set Theory Very Basics

Background

A *set* is a collection of things; this can be anything: numbers, letters, bugs, sources of angst, whatever.

Sample Sets

$A = \{ 1, -18, 47, 12 \}$

$B = \{ a, b, \{ a, c, f \}, x \}$

$C = \{$ dragonfly, giant spider, fire ants $\}$

Teminology & Notation

- The members of a set are called its *elements* or *members*.
- Capital letters are used to denote a set; lower case letters denote members of a set; thus:

  $A = \{ a,b,c,d,e \}$
- Elements are *distinct;* that is, each appears only once.
- Symbols:

  $b \in A$ — b is an element of A

  $x \notin A$ — x is not an element of A

  $\varnothing$ — The *empty* set or *null* set; a set with no elements: $\{ \}$
- Sets may be finite or infinite.

  Finite sets: $F = \{ -8,7,2 \}$ $G = \{$ All tables in this room $\}$

  Infinite sets: $H = \{$ the set of all integers $\}$ $J = \{$ Yorick's Jest $\}$ ← Remember Hamlet?
- The element of a set may itself be a set:

  $B = \{ 8, 17, \{2, 3, 5\}, -111.8 \}$

Specifying a set

Roster Notation: Specifying as a List

- Itemize the elements of the set, listed between { braces }

  $A = \{ 1, 12, 45, 17 \}$
- Order doesn't matter.

  $A = \{ 12, 1, 17, 45 \}$ ← this is the same as the previous A.
- Three dots may be used to indicate missing elements whose values are clear from context:

  $H = \{ 1, 2, 3, \ldots 100 \}$
- Appropriate for finite sets. It may be informally used for infinite sets as long as the meaning is clear:

  $J = \{ 1, 2, 3, 4, \ldots \}$

Predicate/Set Builder Notation - Specifying with a rule

- Describe a properties of the elements:

  $N = \{ x \mid x \text{ is an integer and } x < 8 \}$

  $P = \{ z \mid z \text{ is a letter of the Arabic alphabet} \}$

- In this notation, the vertical bar is read "such that":

  "x such that x is an integer and x is less than 8."

- Note the braces that enclose the set's description.

- To be proper, you should include the *type* of the variable x to the left of the vertical bar:

  $N = \{ x \in \mathbb{N} \mid x > 0 \text{ and } x < 8 \}$

Standard Sets

$\mathbb{N}$	Natural numbers	{ 1, 2, 3, ... }
$\mathbb{W}$	Whole numbers	{ 0, 1, 2, 3, ... }
$\mathbb{Z}$	Integers	{ ..., –1, 0, 1, 2, ... }
$\mathbb{R}$	Real numbers	
$\mathbb{Q}$	Rational numbers	
$\mathbb{I}$	Imaginary numbers	
$\mathbb{C}$	Complex numbers	
$\varnothing$	Empty (null) set	

Some Vocabulary

- Two sets are *identical* if and only if ("iff") they have exactly the same elements.

  That is, $A = B$ only iff for every x,

  $x \in A \iff x \in B$ *Note:* the symbol "⇔" means "implies both ways"

  For example: $\{0,2,4\} = \{ x \mid x \text{ is an even non-negative number less than 5} \}$

- Two sets are *disjoint* if they have no elements in common.

- The *cardinality* of a set is the number of distinct elements it has (that is, ignoring duplicates).

  The cardinalities of { 1, 6, 1, 2, 4, 4 } and { 1, 6, 2, 4} are both *4*.

- The *Universal set* is the set of all objects under consideration. This is usually something that is implicit, though it may be stated explicitly to the left of the vertical bar in set builder format.

  $\{ x \in \mathbb{W} \mid x \leq 20 \}$ ← The universal set is explicitly stated to be all whole numbers

  $\{ x \mid x \text{ is a letter in the word "mysterious"} \}$ ← The universal set is implicitly all English letters

Subsets

- Set A is a ***subset*** of set B iff every element of A is also an element of B. This is denoted: $A \subseteq B$

  $\{ a, b \} \subseteq \{ d, a, c, b \}$ $\{ a, b \} \subseteq \{ a, b \}$ $\{ \{2, 7\} \} \subseteq \{ 5, 12, \{2,7\}, 19 \}$

- If $A \subseteq B$ and $A \neq B$, then A is called a ***proper subset*** of B, written:

  $A \subset B$ $\{ a, b \} \subset \{ d, a, c, b \}$ $\{ a, b \} \not\subset \{ a, b \}$ ← note the "not a..." symbol

- Note that the empty set is a subset of every set:

  $\varnothing \subseteq A$ for every set A

  Nonetheless, the empty set is not an element of A unless explicitly listed, as in

  $A = \{ \varnothing, 1, 18, \text{bunny} \}$

- Note also that a set is always a subset of itself, though it is not a *proper* subset of itself.
- If $A \subset B$, then B is a ***superset*** of A.

Operations on Sets

Intersections & Unions

- The ***union*** of two or more sets is a new set containing the combined elements of the original sets; the symbol for union is "$\subset$".

  Formally: $A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$

  $\{ 1, 6, 7, 3, 4 \} \cup \{ 2, 8, 17, 12\ 4, ½ \} = \{ 1, 6, 7, 3, 4, 2, 8, 17, 12, ½ \}$

- The ***intersection*** of two sets is a new set containing the elements that the original sets have in common; the symbol for intersection is "$\cap$".

  Formally: $A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$

  $\{ 1, 6, 7, 3, 4 \} \cap \{ 2, 8, 17, 12\ 4, ½ \} = \{ 4 \}$

  $\{ a, b, c, d \} \cap \{ e, f, g, h \} = \varnothing$ ← The empty set, you remember (of *course*, you do!)

Intersections, Unions, and the Empty Set

- For every set A,
  - ▹ $A \cap \varnothing = \varnothing$
  - ▹ $A \cup \varnothing = A$

Complement, Difference

- The *Complement* of a set is a set of all the elements in the universal set that are not part of the set under discussion.

  If $A = \{ x \mid x$ is a letter in the word "mysterious" $\}$, then the complement of A, denoted $\bar{A}$, A', or A^C, is a set of all the letters that are *not* in the word "mysterious."

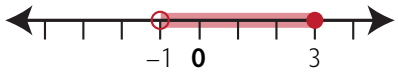
- The *Difference* of two sets is the set of all elements of the first that are not elements of the second.

  $\{ a, b, c, d, e \} - \{ d, e, f \} = \{ a, b, c \}$

  The symbol for difference is "–" or "\"

Interval Notation

- Interval notation is a way of specifying a set consisting of all values between a pair of endpoint values.
- Consider this interval on the real number line:



This interval is written $(-1,3]$

The parenthesis indicates that the low endpoint is not included in the interval.

The bracket indicates that the high endpoint is included in the interval.