

Conversions

Rectangular → Polar

$$x = r \cos(\theta)$$

$$y = r \sin(\theta)$$

Polar → Rectangular

$$r^2 = x^2 + y^2$$

$$\tan(\theta) = \frac{y}{x}$$

Derivatives, Tangent Lines, etc.

Slope

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

← Remember that $x = r \cos \theta$ and $y = r \sin \theta$

← Note that the tangent is horizontal when $dy/d\theta$ is zero and vertical when $dx/d\theta$ is zero

Tangent Lines at the Pole

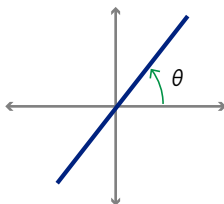
If $f(a) = 0$ and $f'(a) \neq 0$, then the line $\theta = a$ is tangent at the pole to the curve $r = f(\theta)$

Common Curves

Lines

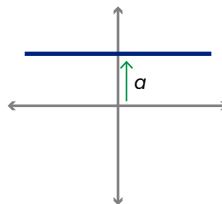
$$\theta = \text{constant}$$

Line through origin



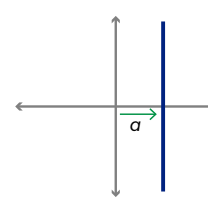
$$r = a \csc(\theta)$$

Horizontal line



$$r = a \sec(\theta)$$

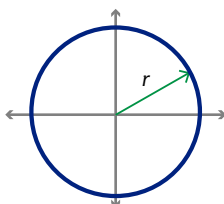
Vertical line



Circles

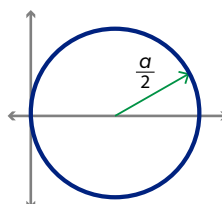
$$r = \text{constant}$$

Circle centered at origin



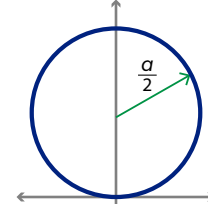
$$r = a \cos(\theta)$$

Circle centered at $(\frac{a}{2}, 0)$



$$r = a \sin(\theta)$$

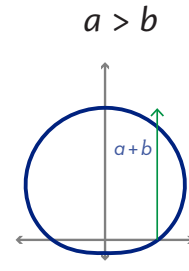
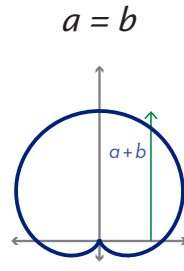
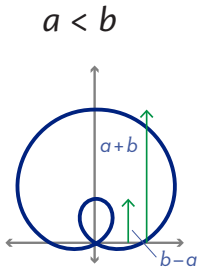
Circle centered at $(0, \frac{a}{2})$



Limaçon

$$r = a \pm b \sin(\theta)$$

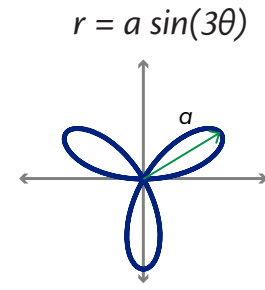
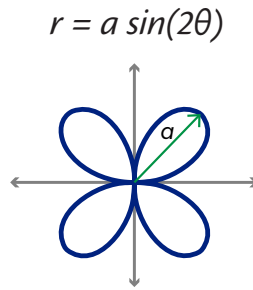
Cosine limaçons extend to the left or right, rather than up or down.



Rose

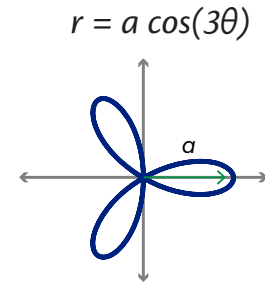
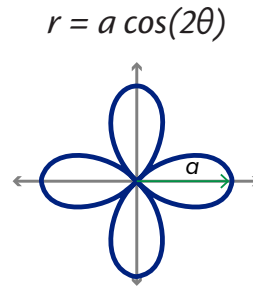
$$r = a \sin(n\theta)$$

- n even: $2n$ petals
- n odd: n petals
- 1st petal starts tangent to the axis



$$r = a \cos(n\theta)$$

- n even: $2n$ petals
- n odd: n petals
- 1st petal starts centered on the axis



Lemniscates

- *sine*: petals aligned 45°
- *cosine*: petals on x-axis

