# Logic and Reasoning

## Inductive vs Deductive Reasoning

### Inductive Reasoning

- Set of specific observations ⟶ general principle
  - ▹ Two-thirds of the students at Whatsamatta U., Hoo R. U, and Howr U. take goat herding, therefore ⅔ of all students everywhere take goat herding.

### Deductive Reasoning

- General principle ⟶ a specific instance
  - ▹ All mammals have ears, therefore Mr. Ed (the talking horse; look it up) has ears.

## Conditional Statements

- A statement of the form "if $p$, then $q$" ($p \longrightarrow q$), where $p$ and $q$ are statements.
  - ▹ If I rob a bank, then I will be arrested.
  - ▹ Every Tuesday I eat snails for lunch. (*i.e.*, If today is Tuesday, then I'll eat snails for lunch.)
- $p$ is the *hypothesis*; $q$ is the *conclusion*.
- A *counterexample* is a specific case in which $p$ is true, but $q$ is not.
  - ▹ Today is Tuesday, but I had barbecued water for lunch.

| Symbols | |
|---|---|
| ⟶ | then, leads to |
| ↔ | if and only if, iff |
| ~ ! ¬ | Not, negation |
| & ∧ ∩ | And, intersection |
| \| ∨ ∪ | Or, union |

### *Derived Statements*

- *Converse*: if $q$, then $p$ ($q \longrightarrow p$)
  - ▹ If I am arrested, then I robbed a bank.
- *Inverse*: if not $p$, then not $q$ ($\sim p \longrightarrow \sim q$)
  - ▹ if I didn't rob a bank, then I won't be arrested.
- *Contrapositive*: if not $q$, then not $p$ ($\sim q \longrightarrow \sim p$)

The contrapositive has the same truth value as the original statement.

  - ▹ If I am not arrested, then I didn't rob a bank.
- *Biconditional*: $p$ if and only if $q$ ($p \longleftrightarrow q$)

"If and only if" is routinely abbreviated "*iff*"

  - ▹ Both the statement ($p \longrightarrow q$) and its converse ($q \longrightarrow p$) are true.
  - ▹ I robbed a bank if and only if I'm arrested.



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## Laws of Logic

*Law of Detachment*

- If $p \longrightarrow q$ is true, and $p$ is true, then $q$ is true.

*Law of Syllogism*

- If $p \longrightarrow q$ is true and $q \longrightarrow r$ is true, then $p \longrightarrow r$ is true.